BI-CO MATHEMATICS COLLOQUIUM

Thomas Church
Stanford University & Institute for Advanced Study

"The cap set problem & fast matrix multiplication"

Monday, February 6, 2017

Talk at 4:00 – H109
Tea at 3:30 – KINSC Math Lounge, H208

Abstract:

Out of the 81 cards in the game of Set, what is the largest number of cards you can put down without forming a set? You can formulate this mathematically as: out of the 81 vectors in the vector space $\mathbf{F}_3^4$, what is the largest subset that does not contain any line? (the answer: 20 cards/vectors) The cap set problem, which has been open for many decades, asks: as n goes to infinity, what is the biggest subset of $\mathbf{F}_3^n$ containing no line? Can it occupy a greater and greater proportion of these vector spaces? Or is there an exponential gap that cannot be crossed? Last summer, five other mathematicians made a huge breakthrough using the "polynomial method": they showed the biggest subsets cannot be larger than 2.8^n. This completely solves the cap set problem.

The most amazing thing about this breakthrough is that the proof is COMPLETELY elementary; all that is necessary is factoring of polynomials, basic linear algebra, and the central limit theorem. In this self-contained talk I will give the entire proof of the solution to the cap set problem. I'll also discuss my own work applying these ideas to computational complexity, on approaches to find the fastest possible algorithm to multiply two matrices. This talk should be accessible to anyone familiar with linear algebra, polynomials, and modular arithmetic.

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